Year 2021 Fall Semester Computational Theory Qualifying Exam (100 points)

1. Convert the NFA shown in Fig. 1 into an equivalent DFA step by step. (10%)
2. Reduce the DFA shown in Fig. 2 into a DFA with the minimum number of states. (10%)
3. Prove that language L is NOT context free by using the closure properties and pumping theory of context free language. $L=\{w_1w_2\},\ w_1\in(a\,|\,b)^*, w_2\in(b\,|\,c)^*$ and the numbers of 'a', 'b' and 'c' are $\#(a)=n, \#(b)=n+1, \text{and}\ \#(c)=n+2,\ n>0.$ (15%)
4. Design a push down automata (PDA) to decide the following language: $L=\{(a\,|\,b)^n c^n (a\,|\,b)^{2n}\}$. (1) Define the PDA, (2) Is your PDA deterministic? Explain it. (A DPDA has exact one transition to perform at each step.) (10%)
5. Design a deterministic Turing machine (DTM) to accept the following language by using machine schema: $L=\{wcwcw\,|\,w\in\{a\}^*, \Sigma=\{a,b,c\}\}$. Please, explain your DTM briefly. (10%)
6. Define the following function by using basic functions:{ *add(x,y)≡x+y*, *sub(x,y)≡x-y*, *is_zero(x)::if(x==0) 1 else 0*}, function composition, and minimalization: $f(x,y)=\dfrac{\log_2 x}{\log_2 y},\ x,y\in Z^+$ and $x,y\geq 2.$ (10%)
7. Prove that the following problem is undecidable by reducing the halting (or non-halting) problem into it: Given a Turing machine M_1, will M_1 halt on all strings of a recursively enumerable language L_2. (10%)
8. Prove that the following problem is NP-complete: G = {V, E} is a graph and V and E are the sets of the vertices and edges. Is there a path starting at vertex v_1, ending at vertex v_2 and containing k distinct vertices? (Reducing the Hamilton cycle problem into this problem.) (15%)
9. Prove that the following problem is NP-hard: There is a polynomial defined as follows:

$$P(x,y,z)=\sum_{i=0}^{n-1} x^{s_i} y^{t_i} z^{r_i},\ s_i+t_i+r_i=3,\ 0\leq s_i,t_i,r_i\leq 3,\ n,s_i,t_i,r_i\in Z^+$$. Does

P(x, y, z) have any integer roots? (Reducing the 3-SAT problem into this one.) (10%)

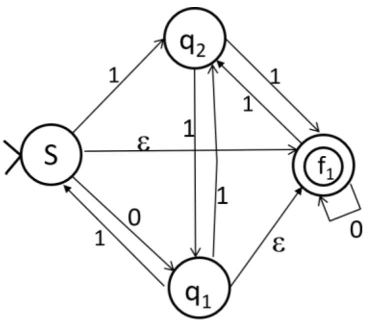

Fig. 1, NFA of Problem 1

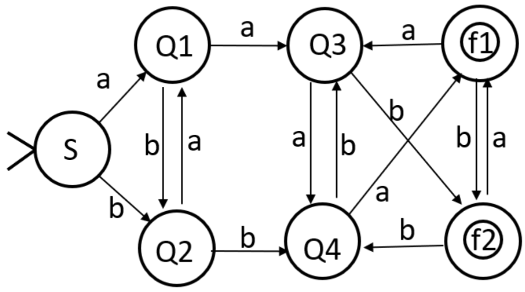

Fig. 2, DFA of Problem 2.