Algorithms
CSE PhD Qualifying Exam
January 2022

1. (20%) Please answer the following questions.

   (a) Show that any binary tree of height h has at most 2^h leaves.

   (b) Illustrate the operation of MAX-HEAP-INSERT$(A, 3)$ on the heap $A = \langle 16, 14, 11, 10, 12, 8, 7, 4, 5, 6, 2, 1\rangle$.

2. (20%) Illustrate the operation of the following sorting algorithms respectively on the array $A = \langle 7, 5, 6, 4, 0, 3, 9, 6, 9, 3, 6\rangle$, where $A[j] \in \{0, 1, \ldots, 9\}$ for $1 \leq j \leq 11$. Which of them are stable sorting algorithms? Which of them are in-place sorting algorithms?

   (a) INSERTION SORT

   (b) COUNTING SORT

3. (20%) A subsequence is a sequence that can be derived from another sequence by deleting some elements. Given two sequences $X = \langle x_1, x_2, \ldots, x_m\rangle$ and $Y = \langle y_1, y_2, \ldots, y_n\rangle$, the longest common subsequence problem is to find a maximum-length common subsequence of X and Y.

   (a) Find an LCS of $\langle D, C, B, A, D, B, A\rangle$ and $\langle B, A, C, D, A, B\rangle$.

   (b) Describe an algorithm that solves the longest common subsequence problem in $O(mn)$ time.

4. (20%) Finding a missing number. An array of n elements contains all but one of the integers from 1 to $n + 1$.

   (a) Give the best algorithm you can for determining which number is missing if the array is sorted, and analyze its asymptotic worst-case running time.

   (b) Give the best algorithm you can for determining which number is missing if the array is "not" sorted, and analyze its asymptotic worst-case running time.

5. (20%) Polynomial-Time Reductions:

   (a) Describe the 3-SAT Problem.

   (b) Describe the INDEPENDENT SET Problem.

   (c) Suppose that 3-SAT Problem is NP-complete, then prove INDEPENDENT SET is also NP-complete.
   Hint:

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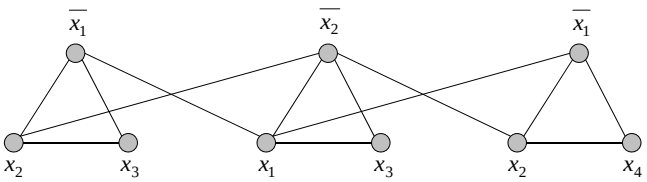

k = 3

$$\Phi = \left(\overline{x_1} \vee x_2 \vee x_3\right) \wedge \left(x_1 \vee \overline{x_2} \vee x_3\right) \wedge \left(\overline{x_1} \vee x_2 \vee x_4\right)$$