國立台灣海洋大學資訊工程學系博士班

109 學年度第二學期演算法資格考命題卷

1. (20%) Please describe briefly the following sorting algorithms along with their time complexities. Which of them are stable sorting algorithms? Which of them are in-place sorting algorithms?

   (a) SELECTION SORT.

   (b) QUICKSORT.

2. (20%) The max-heap property is that for every node i other than the root, $A[\text{PARENT}(i)] \geq A[i]$, that is, the value of a node is at most the value of its parent. Thus, the largest element in a max-heap is stored at the root, and the subtree rooted at a node contains values no larger than that contained at the node itself.

   (a) Is the array with values $\langle 23, 17, 14, 6, 13, 10, 1, 5, 7, 12\rangle$ a max-heap?

   (b) The operation HEAP-DELETE(A, i) deletes the item in node i from heap A. Give an implementation of HEAP-DELETE that runs in $O(\log n)$ time for an n-element max-heap.

3. (20%) Given an array $A = \langle x_1, x_2, \ldots, x_n\rangle$ of numbers, the maximum-subarray problem is to find two indices i and j such that $A = \langle x_i, x_2, \ldots, x_j\rangle$ has the greatest sum of any contiguous subarray of array A.

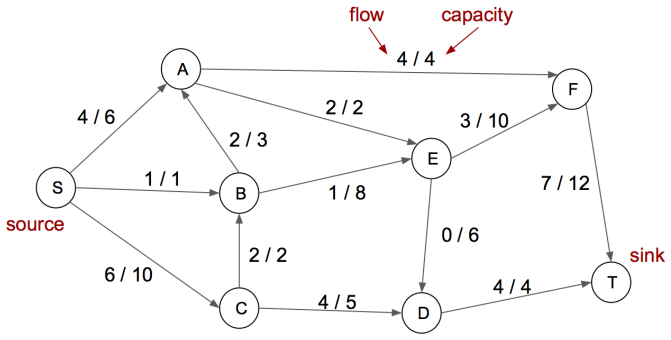
   (a) Illustrate the operation of FIND-MAXIMUM-SUBARRAY on the array $A = \langle -2, 6, -3, 4, -5, 3, 4, -7\rangle$.

   (b) Describe an algorithm that solves the maximum-subarray problem in $O(n \log n)$ time.

4. (20%) The following graph shows a flow network on which an s-t flow has been computed.

   (a) What is the value of this flow? Is this a maximum (s, t) flow in this graph?

   (b) Find a minimum s-t cut in the flow network pictured in the graph, and also say what its capacity is.



5. (20%) Famous ride-sharing company NTOU has created a new "shared-bus" service. The bus serves a path of m miles, indexed by $[0, m]$. The main differences between this service and traditional bus services are:

   - All the passengers board the bus at point 0.
   - Passenger i specifies a range $[s_i, t_i]$, where $0 \leq s_i \leq t_i \leq m$. The passenger is satisfied as long as the bus stops at some location between s_i and t_i (including the boundary).

   There are no fixed bus stops. Instead, the stops will be decided after gathering the user information. As an engineer at NTOU, you need to solve the following problem: given the number of passengers n, and each passenger's preference $[s_i, t_i]$, design an algorithm that finds the smallest number of stops in order to satisfy all customers.

   (a) Design the algorithm and analyze its running time. Your algorithm should run in $O(n \log n)$ time.

   (b) Prove the correctness of the algorithm you designed in (a).