1. (15%) Determine whether each statement in the following is true or false. If the statement is true, prove it. If the statement is false, disprove it or give a counterexample.

   (a) $n^n = O(2^n)$.

   (b) If $f(n) = \Theta(g(n))$, then $2^{f(n)} = \Theta(2^{g(n)})$.

   (c) If $f(n) = O(g(n))$, then $g(n) = \Omega(f(n))$.

2. (20%) It is quite an often problem to select the k-th small item from an array of length n. We of course can first sort the array and then output the answer directly. However, the complexity of sorting is $\Theta(n \lg n)$.

   (a) Write an algorithm for select the k-th small item in linear time for average case.

   (b) Prove that the complexity in average case is $\Theta(n)$.

3. (20%) Write down an algorithm for finding the longest common subsequence of two given sequences. Trace your algorithm for the sequences EASYTEST and TSETYSAE. In this example, one can see that there are more than two longest common subsequences with the same length. If your algorithm can find them all, prove it; otherwise, inspect your algorithm and discuss how to modify the algorithm for obtaining more or even all answers.

4. (15%) What is the worst-case time of *quicksort*? What technique can be applied to avoid worst cases which always select bad partition elements? Give a simple example to show that *quicksort* is unstable. Trace *quicksort* algorithm in nondecreasing order for the input array:

   $$65, 49, 13, 64, 25, 82, 54, 27, 78, 34, 35, 17, 7$$

5. (15%) Find the minimum number of scalar multiplications to compute the matrices
   $$M_{3,2}\ M_{2,5}\ M_{5,4}\ M_{4,6}\ M_{6,3}.$$

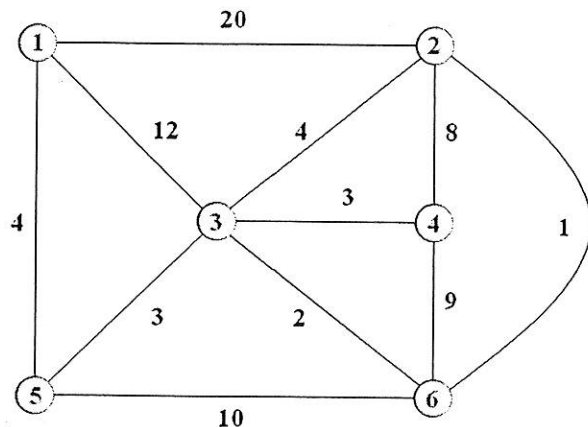6. (15%) Trace Floyd's algorithm to find the shortest path between each pair of vertices for the graph of Figure 1.

Figure 1