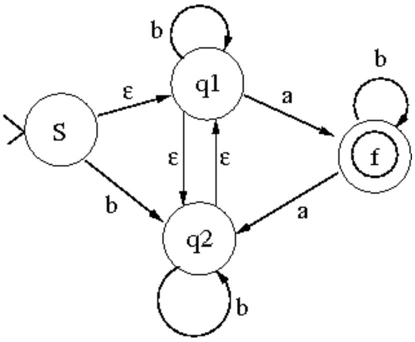
2013 Computational Theory Qualifying Examination

1. (a) Convert the following NFA into a DFA. (b) Then convert the resulted DFA into a DFA with the minimum number of states. (15%)



2. Language **L={a^nba^n | n>0}**. Prove that this language is not regular by using (15%)
   a、 Pumping theorem of regular language.
   b、 MyHill-Nerode theorem.

3. Please complete the following problems: (15%)
   a、 Design a Deterministic Push-Down Automata (DPDA) to accept the following language: **L={a^nba^{2n}| n>0}**. Please verify the correctness of your DPDA by giving explanation of the algorithm and an example. (Reminder: your PDA must be deterministic.)
   b、 Write down the Context Free Grammar (CFG) of the language L.
   c、 Is L un-ambiguous? Please verify it.

4. Prove that the following language is not context free by using the Pumping theory and closure properties of context free language: Σ={a,b,c}, **L={w|#(a)=#(c)<#(b)}**. That is the numbers of 'a' and 'c' in **w** are equal. But the number of 'b' is greater. (10%)

5. Define the following functions by using basic functions, function composition, recursive definition and minimalization. Assume the following basic functions have been defined: (15%)
   ***add(x, y)* ≡ *x+y*, *mul(x, y)* ≡ *x*y*, *sub(x, y)* ≡ *x-y*, *div(x,y)*≡*x/y*, *neq(x, y)*≡*x≠y*.**
   a、 e(x, n) $\equiv \sum_{i=0}^{n} \frac{x^i}{i!}$
   b、 $\log(a, b, x) \equiv \log_a \log_b(x)$
   c、 factor(x, y)≡1, if x is a factor of y,
   0, otherwise.

6. Prove that the following problem is unsolvable by using problem reduction. (Please use the unsolvable problems listed in the textbook for the reduction step.)

   The problem: *Given a Turing machine* $\boldsymbol{M}$ *and a string* $\boldsymbol{x}$ *with the length* $|\boldsymbol{x}|<k$*, where k is a positive integer, will* $\boldsymbol{M}$ *halt on* $\boldsymbol{x}$*?* (The input string contains at least one character.) (15%)

7. It is well-known that the 2-Patition problem is NP-hard, prove that the following 3-partition problem is NP-complete: (15%)

   The 3-partition problem: *Given a set* $\boldsymbol{A}=\{x_1, x_2, x_3, \ldots, x_n\}$ *of non-negative integers, can we divide* $\boldsymbol{A}$ *into three subsets* $\boldsymbol{A_1}$, $\boldsymbol{A_2}$, *and* $\boldsymbol{A_3}$ *such that the summation of the integers in each subset is equal to* $(\sum_{i=1}^{n} x_i)/3$*?* ($||\boldsymbol{A_1}||=||\boldsymbol{A_2}||=||\boldsymbol{A_3}||$.)

   The 2-Partition problem: Given a set $S=\{a_1, a_2, a_3, \ldots, a_n\}$ of non-negative integers. K is an integer and $K=(\sum_{i=1}^{n} a_i)/2$. Is there a subset $P=\{c_1, c_2, \ldots, c_m\}$ of S such that $\sum_{j=1}^{m} c_j = K$?