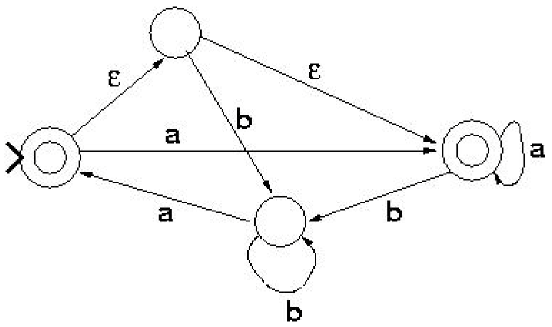
Qualifying Examination, Computational Theory, 2012

1. Prove that the following language is NOT regular by using Pumping lemma and closure properties: $L = \{w = w^R\}$. $\Sigma = \{a,b\}$. (Hint: Intersect L with some regular language and apply Pumping Lemma.) (10%)

2. Convert the following NFA into a DFA.(15%)



3. Prove that the following language is NOT context free by using Pumping theorem: $L = \{a^{n^2}.\}$ (10%)

4. (a) Prove that the following language is context free by designing a Push Down Automata to accept it: $L = \{w = w^R\}$. $\Sigma = \{a,b\}$. (b) Is your PDA deterministic or non-deterministic? Please verify it. (15%)

5. Design a Turing Machine (TM) by using machine schema to accept the following language: $L = \{w = w^R\}$. $\Sigma = \{a,b\}$. (10%)

6. Prove that the following problem is undecidable: M is a Turing machine and w is a string. Will M halts on all substrings of w ? (Hint: Use the halting problem to prove this assertion. The set of the substrings includes w itself.) (10%)

7. Prove that the following problem is undecidable: (15%)
   M1 and M2 are two TMs and they accept L1 and L2 respectively. Is L2 a subset of L1? (Hint: design a special M1, for example, a TM accepts all strings. Then use Rice's theorem to prove this problem. Rice's theorem says that it is undecidable to verify whether the language semidecided by a TM is recursive, context-free, regular,…)

8. It is known that the Hamilton cycle problem is NP-hard. (Given a graph G, is there a cycle passing each node of G exactly once.) Prove that the following problem is NP-complete:

   A. Given a graph G and an arbitrary integer K, is there a cycle passing K nodes of G exactly once? (15%)