國立台灣海洋大學資訊工程學系博士班

98學年度第二學期博士班資格考命題卷（筆試）

科目：演算法
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1. (20%) Please briefly describe the following standard terms and techniques which are commonly used in algorithm designs.

   (a) Approximation Algorithm

   (b) NP-Completeness

   (c) Master Method

   (d) Greedy Method

2. (15%) For a given function $g(n)$, we denote by $O(g(n))$ the set of functions

   $$O(g(n)) = \{\ f(n) \mid \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le f(n) \le cg(n) \text{ for all } n \ge n_0\}.$$

   (a) Show that $n^2 - 9n = O(n^2)$.

   (b) Please show that if $f(n) = a_m n^m + \ldots + a_1 n + a_0$, then $f(n) = O(n^m)$.

3. (15%) A *recurrence* is an equation or inequality that describes a function in terms of its value on small inputs.

   (a) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 5T(\lfloor n/5 \rfloor) + n$.

   (b) Use the substitution method to verify your answer in (a).

4. (15%) Suppose you are given two sets A and B, each containing n positive integers. You can choose to reorder each set however you like. After reordering, let a_i be the ith element of set A, and let b_i be the ith element of set B. You then receive a payoff of $\prod_{i=1}^{n} a_i^{b_i}$.

   (a) Give an algorithm that will maximize your payoff.

   (b) Prove that your algorithm maximizes the payoff, and state its running time.

5. (15%) Please describe briefly the following sorting algorithms along with their time complexities.

   (a) Heapsort

   (b) Quick sort

   (c) Counting sort

6. (20%) A *subsequence* is a sequence that can be derived from another sequence by deleting some elements. Given two sequences $X = \langle x_1, x_2, \ldots, x_m \rangle$ and $Y = \langle y_1, y_2, \ldots, y_n \rangle$, the ***longest common subsequence problem*** is to find a maximum-length common subsequence of X and Y.

   (a) Find an LCS of $\langle A, B, C, B, D, A, B \rangle$ and $\langle B, D, C, A, B, A \rangle$.

   (b) Describe an algorithm that solves the longest common subsequence problem in $O(mn)$ time.