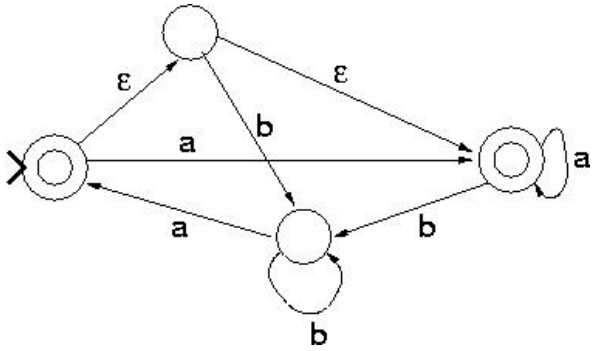
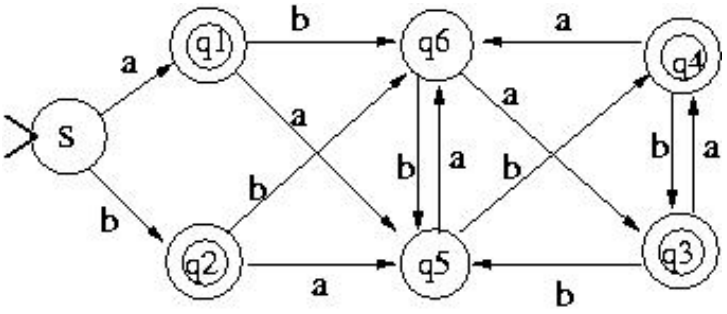
Qualifying Examination, Computational Theory, 2010

1. Convert the following NFA into an equivalent DFA: (10 %)



2. Minimize the following DFA: (10 %)



Minimize the DFA.

3. $L=\{a^n b^m c^k, a+k = m\}$, Prove that L is context free by: (15 %)
   a、 Giving the context free grammar of L.
   b、 Designing a Pushdown Automata for L.
   c、 Is L a Deterministic Context Free Language? Verify your answer, please.
4. $L=\{a^n b^m c^k, m>n \text{ and } m>k\}$. Please prove that L is not context free by using the Pumping theorem. (10 %)
5. L_1 is decidable by some deterministic Turing machine. Prove that L_2 is also decidable by some Turing machine: (Remember that Turing machines = algorithms. Don't hesitate to design an algorithm to solve this problem.)
   a、 $L_2 = \{w|w^R \text{ is a string of } L_1\}$. (15 %)
6. It is known that the following problem is unsolvable:
   {Given a Turing machine M, will M halt on all strings of an alphabet Σ ?}
   Prove the following problem is also unsolvable by using problem reduction:
   a、 Given two machines M_1 and M_2, M_1 decides the language L_1, and M_2 decides the language L_2. Is L_1 a subset of L_2 ? (15 %)

7. Prove that the following problem is NP-complete by reducing the 3 SAT problem to this problem.: (15 %)
   a、 Given a graph G=(V,E) and an integer K, is there a Hamiltonian cycle of K vertices.
8. Prove that the following problem is in both the co-NP and the EXP-classes:
   a、 M_1 and M_2 are two NFA. Is $M_1 = M_2$? (10%)